Optimal Allocation Problem of PV Systems Using Discrete Particle Swarm Optimization with a Hybrid Discretization Scheme

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Abstract—This paper presents a discrete particle swarm optimization (PSO) for the optimal allocation problem of PV systems in distribution power systems. The formulation of the allocation problem for PV systems may include multi-valued discrete variables. Mapping the real-valued particle’s velocity to a multi-valued discrete variable, in the discrete PSO procedure, is performed by a hybrid discretization scheme. The numerical results show the usefulness of the discrete PSO for the allocation problem.

Index Terms—discrete particle swarm optimization, hybrid discretization scheme, optimal allocation, photovoltaic systems

I. INTRODUCTION

The optimal placement and sizing of PV systems in distribution power network is a complex optimization problem. The problem involves discrete and continuous variables. Due to the discrete and discontinuous nature of the problem, classical optimization techniques might be rendered unsuitable and hence the use of global search techniques is justified. Recently, in addition, meta-heuristic optimization methods are widely applied, especially in hard optimization problems involving continuous and discrete variables.

In this paper, a methodology for the optimal allocation of photovoltaic systems in distribution systems is presented, using a discrete PSO algorithm. The discrete PSO is modified from a standard PSO with linearly decreasing inertia weight (PSO-LDIW) by the inclusion of a hybrid multi-level discretization scheme. In order not to compromise the robustness of the PSO algorithm, the same velocity update equation that preserves the social and cognitive components is used but the particle’s position is updated in an alternative way. The variables are not converted into equivalent binary representations; instead, in this paper, a hybrid multi-level discretization scheme is adopted. In the initial iterations of the discrete PSO application the quantization is done by the normal used rounding function, while in the later iterations it is performed by the power-of-two terms boundaries of the transformed velocity using a sigmoid function.

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II. PROBLEM FORMULATION

Assuming that the candidate buses are given for the installation of PV systems and that the maximum capacity of PV systems to be installed at each bus is provided, represented as \( (\Delta P_{uM1}, \Delta P_{uM2}, \ldots, \Delta P_{uMc}) \), where \( c \) denotes the total number of candidate buses, and \( \Delta P \) is the capacity of a PV system considered and \( u_{MM} \) is the maximum integer variable, corresponding to the maximum capacity at bus \( m \). In the problem, it is substantial to determine the appropriate number of units and the best locations in the given network.

To evaluate the operational improvement of the PV installation, active power losses can be taken as the performance index for the PV allocation planning problem. The losses in the network branches can be calculated as the difference of the injected power between the sending end and the receiving end buses. The system loss then can be described as follows:

\[
 f_1 = \sum_{m=1}^{b} P_{loss}^m = \sum_{m=1}^{b} (P_{s,m} - P_{r,m})
\]

where \( P_{s,m} \) and \( P_{r,m} \) denotes the injected active power from the sending and receiving end, respectively, and \( b \) stands for the total number of branches. The objective function is minimizing \( f_1 \).

Equality constraints for nonlinear power flow equations: The power balance equation is included as equality constraints to ensure the balance between supply and demand. These constraints for bus \( m \) are presented as follows:

\[
 0 = P_{inm} + \Delta P_{mH} - P_{Dm} - V_{m} \sum_{j=N} V_{j} \cdot \\
  \left(G_{mj} \cos \theta_{mj} + B_{mj} \sin \theta_{mj}\right)
\]

\[
 0 = Q_{inm} - Q_{Dm} - V_{m} \sum_{j=N} V_{j} \cdot \\
  \left(G_{mj} \cos \theta_{mj} - B_{mj} \sin \theta_{mj}\right)
\]

Voltage levels at the distribution buses should be within the established limits to maintain power quality. This constraint is
described as follows:

\[
V_m^{\min} \leq V_m \leq V_m^{\max} \tag{4}
\]

Inequality constraint associated by the PV systems penetration level: The maximum PV penetration level should be smaller or equal to the specified amount. The maximum PV penetration level can be set as a percentage of the total load. In this paper, the maximum PV penetration level is set to 80% of the total real power load of the network. This constraint is now described as follows;

\[
P_{pv}^{\text{gen}} \leq P_{pv}^{\text{spec.}} \tag{5}
\]

where the following notations are made: \( P_{gen} \) is real power generated at bus \( m \); \( P_{Dnm} \) is real power load at bus \( m \); \( Q_{gen} \) is reactive power generated at bus \( m \); \( Q_{Dnm} \) is the reactive power load at bus \( m \); \( \theta_{ij} \) is the conductance and susceptance for the \( (m, j) \) component from \( Y \) bus matrix; \( \alpha \) is the voltage angle difference between bus \( m \) and \( j \); \( V_m \) and \( V_j \) are voltage magnitudes of bus \( m \) and \( j \), respectively; \( V_m^{\max} \) and \( V_m^{\min} \) are the upper and lower limits for bus \( m \), respectively; \( P_{pv}^{\text{gen}} \) and \( P_{pv}^{\text{spec.}} \) are the total injected power and the specified penetration level of PV units, respectively; \( \Delta P_{pv} u_m \) is the power injected of PV at bus \( m \) with a unit number \( u_m \).

Equations (2) and (3) are the equality constraints for the active and reactive power balance at each bus. In equation (2), \( u_m \) is an integer variable for the number of PV units that are installed at bus \( m \), and \( u_m = 0, ..., u_{\text{max}} \). Voltage deviation at each bus is restricted in the upper and lower limit and described as an inequality constraint in (4). Another constraint included is the inequality constraint described in (5) that limits the total penetration from PV units by the specified power, \( P_{pv}^{\text{spec.}} \). To force these constraints within the limits, these two inequality constraints are converted into the objective function as quadratic penalty terms for application of simulation based optimization techniques such as PSO. Then, the augmented objective function can now be described as:

\[
\min F = f_1 + \alpha_1 f_2 + \alpha_2 f_3
\]

\[
f_2 = \begin{cases} 
0 , & P_{gen}^{pv} \leq P_{pv}^{spec} \\
1 , & P_{gen}^{pv} > P_{pv}^{spec} 
\end{cases} \tag{6}
\]

\[
f_3 = \sum_{m=\text{av}} \left( V_m^{\max} - V_m \right)^2 + \sum_{m=\text{lv}} \left( V_m^{\min} - V_m \right)^2
\]

where \( \alpha_1 \) and \( \alpha_2 \) are the penalty factors against the solution’s violating the two inequality constraints, and \( \text{av} \) and \( \text{lv} \) are the sets of buses whose voltage magnitudes violating the upper and lower voltage limits, respectively. If the set of buses exceeds beyond statutory voltage limits, in this paper, they are penalized by a very high value of \( \alpha_1 \left(10^6\right) \) for the unregulated voltage deviation at\( \alpha \)-th bus.\( \alpha \) is the value introduced if the PV penetration level exceeds beyond the maximum penetration value. For this purpose, the injection violation index, \( f_2 \), is employed. Normally, \( f_2 \) is set to 0 but it is set to 1, if \( P_{gen}^{pv} > P_{pv}^{spec.} \). This optimization problem is severely nonlinear due to the equality constraints of (2) and (3) and the original objective function \( f_3 \), which is said to be having lots of local minima. Besides, the main variables of the problem are integer as the penetration level of PV system at a candidate bus \( m \) is expressed by \( \Delta P_{pv} u_m \).

III. HYBRID MULTI-LEVEL DISCRETIZATION

The discrete PSO, adopted in this paper, is a variant modified from a real-coded standard PSO with linearly decreasing inertia weight. To adequately deal with the discrete variables, thus, a certain quantization method should be performed to map the real-valued velocity of each particle to an adequate integer. In the literature, there are several ways to transform real-valued particles’ velocities into discrete ones for discrete PSOs [1-7].

This paper indeed employs a hybrid multi-level quantization scheme. The motivation of applying the hybrid discretization comes from the fact that in the initial PSO iterations global exploration needs to be emphasized for the solution not to get trapped into local minima and that in the later iterations local search would be better to facilitate the convergence. According to the motivation, this paper first applies a simple rounding function for the discretization of particles’ velocities as follows:

\[
\Delta V_j^{i+1} = \begin{cases} 
0 , & V_j^{i+1} \leq V_j^{k+1} \\
1 , & V_j^{i+1} > V_j^{k+1} 
\end{cases}
\]

\[
f_j = \begin{cases} 
V_j^{i+1} \leq V_j^{k+1} \\
V_j^{i+1} > V_j^{k+1} 
\end{cases}
\]

where \( V_j^{i+1} \) is the velocity of the \( j \)-th variable of the particle at \( j+1 \) iteration and \( k_{\text{max}} \) and \( k_{\text{min}} \) are the maximum and minimum velocities to force the particle’s position within the feasible range.

After a certain point of the discrete PSO, it switches the discretization scheme to another one based on a sigmoid function. For this purpose, the values of velocity of particles are mapped to a hypercube with the range of (-1, 1) using the sigmoid function [8] as follows:

\[
S_j = \text{sig}(V_j^{k+1}) = -1 + \frac{2}{1 + \exp\left(-V_j^{k+1}/\zeta\right)} \tag{8}
\]

where \( \zeta \) is the steepness of the sigmoid function. Then the set of array of the multi-level discrete values is defined by the quantization level. As seen in Fig. 1, to set for each range, the powers-of-two terms is used as the boundaries to discretize the transformed particles velocities, \( S_j \).

The new position update of the particle is defined by the following rule:
IV. NUMERICAL RESULTS

The hybrid discretization scheme for the discrete PSO algorithm, modified from a continuation version of PSO with LDIM, is applied to the PV allocation optimization problem, to verify its feasibility. The PV system allocation problem is in the category of mixed integer nonlinear programming (MINLP). However, all the continuous variables are included in the objective function as in (6). When applying PSO the objective function values for the particles’ positions are evaluated by the external module, so during the iteration PSO needs to only deal with discrete variables for decision making on the installation of PV units. A modified IEEE-37 bus distribution system is taken as the test system to obtain the numerical experiences on the discrete PSO.

V. CONCLUSIONS

This paper presents a discrete particle swarm optimization (PSO) with a hybrid discretization scheme. The discrete PSO can be applied to optimal allocation problems for multi-units to different locations. This paper mainly considers the PV system allocation problem in distribution power systems, and the problem considers the objective function of loss minimization and the constraints for maximum PV penetration level and voltage profiles in the system. The allocation problem includes multi-valued discrete variables, so to obtain solutions the algorithm needs to perform the mapping the real-valued particle’s velocity to a multi-valued discrete variable by the hybrid discretization scheme, combining a simple rounding function and power-of-two term based boundaries based on a sigmoid function.

REFERENCES